**Al-Farabi Kazakh National University**

**Faculty of Mechanics and Mathematics**

**Department of Differential Equations and Control Theory**

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|  | APPROVED by **Dean of Faculty**  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ D.B.Zhakebaev  (signature)  "\_\_\_" \_\_\_\_\_\_\_\_\_2019 |

### EDUCATIONAL-METHODICAL COMPLEX OF DISCIPLINE

### « Calculus of variations and optimization methods»

**6B060100 – Mathematics**

Course – 4

Semester – 7

Number of credits – 3

**Almaty 2019**

Educational-methodical complex of the discipline is made by Simon Serovajsky, doctor of science, professor

Based on the classing curriculum on the specialty 6B060100 – Mathematics

Considered and recommended at the meeting of the department of Differential Equations and Control Theory

on “\_\_\_” \_\_\_\_\_\_\_\_\_, 2019, protocol № \_\_

Head of the department\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_Kh.Khompysh

(signature)

### Recommended by the methodical bureau of the faculty

on “\_\_\_” \_\_\_\_\_\_\_\_\_, 2019, protocol № \_\_

Chairman of the methodical bureau

of the faculty\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_G.E.Abduakhitova

(signature)

**Syllabus**

**By Educational Program «-------------------»  
…. Semester …. Academic year**

Academic course information

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Discipline’s code | Discipline’s title | | IWS |  | | | | Number of credits | | IWST |
| Lect. | Pract. | | Lab. |
|  | Calculus of variations and optimization methods | |  | 2 | 1 | | - | 3 | |  |
| Lecturer | | Simon Serovajsky, doctor of science, professor | | | | Office hours | | | Scheduled | |
| e-mail | | [serovajskys@mail.ru](mailto:serovajskys@mail.ru) | | | |
| Telephone number | | +7 701 8315197 | | | | Auditory | | |  | |
| Assistant | | Full name, academic degree, academic rank. | | | | Office hours | | |  | |
| e-mail | | E-mail: | | | |
| Telephone number | | Telephone: | | | | Auditory | | |  | |

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| --- | --- |
| Academic presentation of the course | **Aim of course:** Analysis of general methods of calculus of variations and optimization control theory  **As a result of studying the discipline, students should be able to:**   1. To know the applications of the extremum theory; 2. To know the classification of the problems of the extremum theory; 3. To be able to analyze the extremum of functions; 4. To use the variational method for solving problems of minimization of integral functionals; 5. To know the problem statements of optimal control problems; 6. To know general optimization methods; 7. To know approximation methods for optimal control problems. |
| Prerequisites | Mathematical analysis, functional analysis, differential equations, numerical methods |
| Post requisites | Special courses |
| Information resources | **literature**:   1. Эльсгольц Л.Э. Дифференциальные уравнения и вариационное исчисление. – М., Наука, 1969. 2. Будылин А.М. Вариационное исчисление. – Санкт-Петербург, СПбГУ, 2001. 3. Васильев Ф.П. Методы оптимизации. В двух томах. – М.: МЦНМО, 2011. 4. Лутманов С.В. Курс лекций по методам оптимизации. – Ижевск, 2001. 5. Зеликин М.И. Оптимальное управление и вариационное исчисление. – М., Ленард, 2017. 6. Kirk D. E. Optimal Control Theory: An Introduction. – New Jersey, Englewood Cliffs, 2004. 7. Cassel, Kevin W.: Variational Methods with Applications in Science and Engineering, Cambridge University Press, 2013. 8. Snyman, J.A.; Wilke, D.N. Practical Mathematical Optimization : Basic Optimization Theory and Gradient-Based Algorithms (2nd ed.). Berlin: Springer, 2018. 9. Serovajsky, S. Practical Course of the Optimal Control Theory with Examples. Almaty, Қазақ университеті, 2011. 10. Serovajsky, S. Differentiation and Optimization. – London, CRS Press, 2018.   **Internet-resources:**  1. [Dacorogna, Bernard](https://fr.wikipedia.org/wiki/Bernard_Dacorogna). [Introduction to the Calculus of Variations (3rd Edition)](http://www.worldscientific.com/worldscibooks/10.1142/p967), 2014, World Scientific Publishing.  2. [http://www.newlibrary.ru/book/budylin\_a\_m\_/variacionnoe\_ischislenie.html](http://www.newlibrary.ru/book/budylin_a_m_/variacionnoe_ischislenie.html%20) . |
| Academic policy of the course in the context of university moral and ethical values | **Academic Behavior Rules:** Obligatory attendance of classes, intolerance for being late, commitment to deadlines for completion and delivery of assignments (CDS, Practical classes, midterm exams, individual projects).  **Academic values:** According to Article 5 of the Code of Honor of students of Al-Farabi Kazakh National University, a student must strictly fulfill his academic duties and prevent academic and legal violations (plagiarism, forgery, use of cribs, deceit of and disrespectful attitude to teaching stuff, absenteeism and coming late without respectful reasons).  All students can receive counseling assistance in person, by phone at the numbers indicated or by e-mail provided. |
| Evaluation and attestation policy | **Criteria-based evaluation:** evaluation of achieving learning outcomes in accordance with the descriptors (checking competencies acquired at weeks of the intermediate control, midterm and final examinations)  **Summative evaluation:**  Final score of the discipline =  IC1, IC2, are intermediate controls, МТ is Midterm, FE – final exam.  Percent-rating letter system for assessing of achievements of leaning outcomes by students:  95% - 100%: А 90% - 94%: А- 75% - 79%: В-  85% - 89%: В+ 80% - 84%: В 60% - 64%: С-  70% - 74%: С+ 65% - 69%: С 25% -49%: FX  55% - 59%: D+ 50% - 54%: D- 0% -24%: F |
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**Calendar (schedule) the implementation of the course content:**

|  |  |  |  |
| --- | --- | --- | --- |
| **Week / date** | **Topic title (lectures, practical classes, laboratory classes,** **Independent class of students)** | **Number of hours** | **Maximum score** |
| **1** | **2** | **3** | **5** |
| 1 | **Lecture 1. Practical examples of the extremum problems**. Maximization of the flight of the body. Brachistochrone problem. Maximization of the flight of the missile. | 2 | 5 |
| **Practical class 1**. Practical examples of the extremum problems. | 1 | 15 |
| Laboratory class 1. |  |  |
| 2 | **Lecture 2. Minimization of functions**. Stationary condition. Examples. Maximization of the flight of the body. Minimization of the function of many variables. | 2 | 5 |
| **Practical class 2**. Minimization of functions and stationary condition. | 1 | 15 |
| Laboratory class 2. |  |  |
| 3 | **Lecture 3. Euler equation for Lagrange problem.** Lagrange problem. Euler equation. Examples. The fall of the body. Fermat principle and the refraction of light law. | 2 | 5 |
| **Practical class 3**. Euler equation for Lagrange problem. | 1 | 15 |
| IWST. Colloquium (orally). |  |  |
| 4 | Lecture 4. |  |  |
| **Lecture 4. Lagrange problem for the functions family.** Problem statement. The system of Euler equations. Example. Principle of the least action. | 2 | 5 |
| **Practical class 4**. Lagrange problem for the functions family. | 1 | 15 |
| 5 | **Lecture 5. Lagrange problem with high derivatives.** Problem statement. Euler – Poisson Equation. Example. Bending of the elastic beam. | 2 | 5 |
| **Practical class 5**. Lagrange problem with high derivatives. | 1 | 15 |
| Laboratory class 5. |  |  |
| **INTERMEDIATE CONTROLS 1.** | |  | **100** |
| 6 | **Lecture 6. Lagrange Problem for functions with many variables.** Problem statement. Ostrogradsky equation. Dirichlet integral. The oscillation of the string. | 2 | 5 |
| **Practical class 6.** Lagrange Problem for functions with many variables. | 1 | 15 |
| Laboratory class 6. |  |  |
| 7 | **Lecture 7. Bolza Problem.** Problem statement. Necessary conditions of extremum. Transversality conditions. Example. River crossing problem. | 2 | 5 |
| **Practical class 7.** Bolza Problem. | 1 | 15 |
| Laboratory class 7. |  |  |
| IWST. Colloquium (orally). |  |  |
| 8 | **Lecture 7. Bolza Problem.** Problem statement. Necessary conditions of extremum. Transversality conditions. Example. River crossing problem. | 2 | 5 |
| **Practical class 7.** Bolza Problem. | 1 | 15 |
| Laboratory class 8. |  |  |
| 9 | **Lecture 9. Variational problems with pointwise constraints.** Problem statement. Lagrange multipliers method. Example. Oscillation of the pendulum. | 2 | 5 |
| **Practical class 9.** Variational problems with pointwise constraints. | 1 | 15 |
| Laboratory class 9. |  |  |
| IWST. Submission of IWS 2. «Theme» Control class. |  |  |
| 10 | **Lecture 10. Easiest optimization control problems.** Maximization of the flight of the missile (problem statement). Pontyagin’s maximum principle. Example. Iterative method for solving the optimality conditions. | 2 | 5 |
| **Practical class 10.** Easiest optimization control problems. | 1 | 15 |
| Laboratory class 10. |  |  |
| **INTERMEDIATE CONTROLS. (MIDTERM)** | |  | **100** |
| 11 | **Lecture 11. Optimization control problems for the vector case.** Problem statement. Pontyagin’s maximum principle. Example. Maximization of the flight of the missile (solving). | 2 | 5 |
| **Practical class 11.** Optimization control problems for the vector case. | 1 | 15 |
| Laboratory class 11. |  |  |
| IWST. Colloquium (orally). |  |  |
| 12 | **Lecture 12. Optimization control problem with fixed final state.** Problem Statement.Maximum principle. Example. Time optimization problem. Firing method. | 2 | 5 |
| **Practical class 12.** Optimization control problem with fixed final state. | 1 | 15 |
| Laboratory class 12. |  |  |
| 13 | **Lecture 13. Differentiation of functionals and abstract optimization problems.** Gradient methods for functions. Gateau derivatives of functionals. Examples. Gradient methods for functionals. | 2 | 5 |
| **Practical class 13.** Differentiation of functionals and abstract optimization problems. | 1 | 15 |
| Laboratory class 13. |  |  |
| IWST. Colloquium (orally). |  |  |
| 14 | **Lecture 14**. **Variational inequalities**. Variational inequalities and constraints minimization of functional. Examples. Variational inequalities and constraints minimization of functional. | 2 | 5 |
| **Practical class 14.** Variational inequalities. | 1 | 15 |
| Laboratory class 14. |  |  |
| 15 | **Lecture 15. Existence and uniqueness of extremum problems.** Existence theorem for abstract optimization problems. Uniqueness theorem for abstract optimization problems. Example. | 2 | 5 |
| **Practical class 15.** Existence and uniqueness of extremum problems. | 1 | 15 |
| IWST. Submission of IWS 3. «Theme» Control class. |  |  |
| **INTERMEDIATE CONTROLS 2.** | |  | **100** |
| **EXAM** | |  | **100** |
| **GENERAL** | |  | **100** |
| *Note:* IWST *is planned in the amount of 7 hours per semester, points are not put, only the type of class is written (examination, Colloquium).* IWS *is planned in the amount of 3 hours per semester, points must be weight, not less than 20 points. All the CDs it must be the name of the theme. Points for the lecture are not put.* | | | |

Head of the Department of DE and CT KH.KHompysh

Chairman of the Faculty Methodical Bureau G.E.Abduakhitova

Lecturer S. Serovajsky